

Direct Comparison Test

$$A) \sum_{n=1}^{\infty} \frac{1}{n^2 + 9}$$

$$B) \sum_{n=1}^{\infty} \frac{n}{n^2 - 9}$$

$$C) \sum_{n=1}^{\infty} \frac{n}{n^2 + 9}$$

Limit Comparison Test

A) $\sum_{n=1}^{\infty} \frac{n}{n^2+9}$

B) $\sum_{n=1}^{\infty} \frac{n}{n^2-9}$

Diverges

Compare to $\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic Diverges

$$\lim_{n \rightarrow \infty} \frac{\left(\frac{n}{n^2-9}\right)}{\left(\frac{1}{n}\right)} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2-9} = 1$$

$$\lim_{n \rightarrow \infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{n}{n^2-9}\right)} = \lim_{n \rightarrow \infty} \frac{n^2-9}{n^2} = 1$$

D) $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

Converges

C) $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

Compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$
 $p=2 > 1$ converges

$$\lim_{n \rightarrow \infty} \frac{\left(\frac{1}{n^2+4}\right)}{\left(\frac{1}{n^2}\right)} = 1$$

the 2 series do the same thing

converges, E) $\sum_{n=1}^{\infty} \frac{5+3^n}{4+5^n}$

Compare to $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$

$r = \frac{3}{5} \quad |r| < 1$ converges

$$\lim_{n \rightarrow \infty} \frac{\frac{5+3^n}{4+5^n}}{\frac{3^n}{5^n}} = \lim_{n \rightarrow \infty} \frac{5^n(5+3^n)}{3^n(4+5^n)} = \frac{15^n}{15^n} = 1$$

$$5^n \cdot 5^n = 5^{2n} = 25^n$$

What you'll Learn About
Root Test

A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{10^n}$

B) $\sum_{n=1}^{\infty} \frac{(1)}{n^n}$

C) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+10} \right)^n$

D) $\sum_{n=1}^{\infty} \left(\frac{5n}{3n+10} \right)^{2n}$

E) $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+10} \right)^n$